Evaluate $\int \frac{5x^2-20}{x^3-4x^2+20x}\,dx$.

$$x^3-4x^2+20x = x(x^2-4x+20)$$
$$= x\left((x-2)^2+16\right)$$

**SCORE: _____ / 8 PTS**

1 1 1 1

$$= \int \left(\boxed{\frac{A}{x} + \frac{B(2x-4)+C}{(x-2)^2+16}}\right)dx = \int\left(\boxed{\frac{-1}{x}} + \frac{\boxed{3(2x-4)}\boxed{+8}}{(x-2)^2+16}\right)dx$$

$$\boxed{5x^2-20 = A[(x-2)^2+16] + B(2x-4)x + Cx} \quad 1$$

$x=0$: $\quad -20 = 20A \longrightarrow A = -1$

$x=2$: $\quad 0 = 16A + 2C \longrightarrow C = -8A = 8$

COEF OF $x^2$: $\quad 5 = A + 2B \longrightarrow B = \frac{1}{2}(5-A) = 3$

$$= -\ln|x| + 3\ln|x^2-4x+20| + \frac{8}{4}\tan^{-1}\frac{x-2}{4} + C$$

SANITY CHECK:

$x=3$

$$\boxed{\begin{array}{c} \frac{45-20}{27-36+60} \stackrel{?}{=} -\frac{1}{3} + \frac{3(2)+8}{17} \\ \frac{25}{51} \stackrel{?}{=} -\frac{1}{3} + \frac{14}{17} = \frac{-17+42}{51} \end{array}} \quad 1 \quad \checkmark$$

$\frac{1}{2}$

$$= \boxed{-\ln|x|}$$
$$\frac{1}{2} \quad \boxed{+3\ln|x^2-4x+20|}$$
$$1 \quad \boxed{+2\tan^{-1}\frac{x-2}{4}} + C$$

Evaluate $\int_0^5 \frac{12-3z}{z^2-8z+12}\,dz$.

$z^2-8z+12=(z-2)(z-6)$

**SCORE: _____ / 7 PTS**

$$= \int_0^2 \frac{12-3z}{z^2-8z+12}\,dz + \int_2^5 \frac{12-3z}{z^2-8z+12}\,dz$$

2

$$= \lim_{N\to 2^-} \int_0^N \frac{12-3z}{z^2-8z+12}\,dz$$

1

$$= \lim_{N\to 2^-} \left(-\frac{3}{2}\ln|z^2-8z+12|\right)\Big|_0^N$$

1

$$= \lim_{N\to 2^-} -\frac{3}{2}\left(\ln|N^2-8N+12|-\ln 12\right)$$

1

$$-\frac{3}{2}(-\infty-\ln 12)\to\infty$$

1

$$\int \frac{12-3z}{z^2-8z+12}\,dz$$

$$= -\frac{3}{2}\int \frac{1}{u}\,du \qquad u=z^2-8z+12$$

$$du=(2z-8)\,dz$$

$$= -\frac{3}{2}\ln|u| \qquad -\frac{3}{2}du=(12-3z)\,dz$$

$$= -\frac{3}{2}\ln|z^2-8z+12|$$

INTEGRAL DIVERGES 1

Evaluate $\int_0^\infty y^2 e^{-\frac{y}{2}} dy$.

**SCORE: _____ / 7 PTS**

$$= \boxed{\lim_{N\to\infty} \int_0^N y^2 e^{-\frac{y}{2}} dy}$$ 1

$$= \boxed{\lim_{N\to\infty} \boxed{-(2y^2+8y+16)e^{-\frac{y}{2}}}\Big|_0^N}$$ 1 2

$$= \boxed{\lim_{N\to\infty} \left(-(2N^2+8N+16)e^{-\frac{N}{2}}+16\right)}$$ 1

$$= -0+16$$

$$= \boxed{16}$$ $\frac{1}{2}$

$$\lim_{N\to\infty} \frac{2N^2+8N+16}{e^{\frac{N}{2}}} \quad \frac{\infty}{\infty}$$

$$= \boxed{\lim_{N\to\infty} \frac{4N+8}{\frac{1}{2}e^{\frac{N}{2}}}} \quad \frac{\infty}{\infty}$$ $\frac{1}{2}$

$$= \boxed{\lim_{N\to\infty} \frac{4}{\frac{1}{4}e^{\frac{N}{2}}}} \quad \frac{4}{\infty}$$ $\frac{1}{2}$

$$= \boxed{0}$$ $\frac{1}{2}$

| $u$ | | $dv$ |
|---|---|---|
| $y^2$ | $+$ | $e^{-\frac{y}{2}}$ |
| $2y$ | $-$ | $-2e^{-\frac{y}{2}}$ |
| $2$ | $+$ | $4e^{-\frac{y}{2}}$ |
| $0$ | | $-8e^{-\frac{y}{2}}$ |

$$\int y^2 e^{-\frac{y}{2}} dy$$

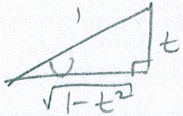
$$= -(2y^2+8y+16)e^{-\frac{y}{2}}$$

Evaluate $\int e^{\arcsin t}\,dt$. **SCORE: _____ / 8 PTS**

EITHER VERSION OK

1 $\boxed{\begin{aligned} u &= \arcsin t \\ t &= \sin u \end{aligned}}$ $\rightarrow$

$dt = \cos u\, du$




| $u$ | $dv$ |
|---|---|
| $\cos u$ | $+\ e^u$ |
| $-\sin u$ | $-\ e^u$ |
| $-\cos u$ | $+\ e^u$ |

1 $\boxed{\int e^u \cos u\, du} = \boxed{e^u \cos u + e^u \sin u - \int e^u \cos u\, du}$ 3

$2\int e^u \cos u\, du = e^u \cos u + e^u \sin u$

$\int e^u \cos u\, du = \boxed{\frac{1}{2} e^u \cos u + \frac{1}{2} e^u \sin u} + C$ 1

$= \boxed{\frac{1}{2} e^{\arcsin t}\sqrt{1-t^2}} + \boxed{\frac{1}{2} t e^{\arcsin t}} + C$

1 1